CLUSTER UNIVERSITY :: KURNOOL

UG SECOND YEAR B.Sc IV SEMESTER EXAMINATIONS AUGUST 2023

SUBJECT : MATHEMATICS-IV

PAPER TITLE : **C4310-REAL ANALYSIS**

Time : 3 Hours Max. Marks : 75

SECTION -A

Answer any **FIVE** of the following questions. Marks : **5 x 5= 25M**

1. Prove that the sequence $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \cdots + \frac{1}{n+n}$ is convergent.
2. Prove that -Every convergent sequence is Cauchy's sequence.
3. Test for convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^2} - n\right)$.
4. Prove that $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - \frac{1}{1\,!} + \frac{1}{2\,!} - \frac{1}{3\,!} + \cdots$ converges
5. Discuss the Continuity of $f(x) = \frac{e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}$
6. Prove that- If $f : [a,b] \to R$ is derivable at $c \in [a,b]$, then f is continuous at c.
7. Find the c of the Lagrange's theorem of $f(x) - (x-1)(x-2)(x-3)$ on $[0,4]$.
8. If $f(x) = x^2$ on $[0,1]$ and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$, then compute $L(P,f)$ and $U(P,f)$.

SECTION -B

Answer the following questions Marks : **5 x 10= 50M**

9. (a) Prove that-"A sequence is convergent if and only if it is bounded and has only one limit point"

(OR)

(b) Apply Cauchy's general principle of convergence to show that the sequence $\{a_n\}$ where $a_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots + \cdots + \frac{1}{n^2}$ converges.

10. (a) State and prove Alembert's ratio test for infinite series.

(OR)

(b) Examine the convergence of $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1} - \sqrt{n}\right)$

11. (a).Let $f; R \to R$ be such that $f(x) = \frac{\sin(a+1)x + \sin x}{x}$ for $x < 0$, $f(x) = c$ for $x = 0$ and

$f(x) = \frac{(x+bx^2)^{\frac{1}{2}} - x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}$ for $x > 0$. Determine the values of a, b, c for which the function is continuous at $x = 0$.

(OR)

(b) State and prove "Intermediate value theorem"

12. (a) State and Prove "Roll's theorem"

(OR)

(b) Find c of Cauchy's Mean mean -value theorem for $f(x) = \sqrt{x}$ and $g(x) = {}^{1}/_{\sqrt{x}}$ in $[a, b]$ where $0 < a < b$.

13. (a). State and prove "the necessary and sufficient condition for integrability"

(OR)

(b). Show that $f(x) = 3x + 1$ is integrable on $[1, 2]$ and $\int_1^2 (3x + 1)dx = \frac{11}{2}$

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